

8-2 The Unit Circle and Finding Exact Value Notes

A **unit circle** is a circle with a radius of 1 and centered at $(0, 0)$ and has equation of $x^2 + y^2 = 1$

- **reference angle** → an acute angle formed between a drawn angle θ and the x-axis.
- **terminal point** → a point (x, y) that falls on the Unit Circle.
- **cosine function** → represents the x -coordinate of the terminal point of an angle on the Unit Circle.
- **sine function** → represents the y -coordinate of the terminal point of an angle on the Unit Circle.

Unit Circle Information:

$$\left. \begin{array}{l} 30^\circ \text{ and } 210^\circ \\ 150^\circ \text{ and } 330^\circ \end{array} \right\} \text{Ref Angle} = 30^\circ$$

$$\left. \begin{array}{l} 45^\circ \text{ and } 225^\circ \\ 135^\circ \text{ and } 315^\circ \end{array} \right\} \text{Ref Angle} = 45^\circ$$

$$\left. \begin{array}{l} 60^\circ \text{ and } 240^\circ \\ 120^\circ \text{ and } 300^\circ \end{array} \right\} \text{Ref Angle} = 60^\circ$$

1. Degrees will be on the inner circle.
2. Radians will be on the middle circle.
3. Terminal point will be on the outer circle ($x = \cos \theta$ and $y = \sin \theta$)

Using our special right triangles (45 – 45 – 90 and 30 – 60 – 90), we know that:

θ	30° or $\frac{\pi}{6}$	45° or $\frac{\pi}{4}$	60° or $\frac{\pi}{3}$	Where is it positive?
$\sin \theta$				
$\cos \theta$				
$\tan \theta$				

Example 1: Using your Unit Circle Sheet, answer each question.

a. What is the reference angle for the angle of 240° ?	b. What is the reference angle for the angle of $\frac{3\pi}{4}$?	c. What is the reference angle for the angle of -750° ?
d. What is the terminal point for the angle of 510° ?	e. What is the terminal point for the angle of $-\frac{9\pi}{4}$?	f. If you are at terminal point $(0, 1)$ and move 300° CCW, what angle did you stop at that is on the UC?

Steps to Find Exact Value of an Angle: Some answers contain radicals/fractions (**NO decimal answers**)

1. Make sure your angle is between 0° and 360° or between 0 and 2π . If it is not, add/subtract 360° or 2π .
2. Locate the correct angle on the Unit Circle. Look at the terminal point if finding the sine or cosine.
3. If finding one of the others, use the equations below:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}; \quad \csc \theta = \frac{1}{\sin \theta}; \quad \sec \theta = \frac{1}{\cos \theta}; \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

* If finding the exact value of a quadrant angle (90° , 180° , 270° , or 360°) \rightarrow use values in terminal points

Example 2: Using your Unit Circle, find the exact value. Remember – NO DECIMALS!!!!

a. $\sin 135^\circ =$ _____	b. $\csc 210^\circ =$ _____	c. $\cos 450^\circ =$ _____	d. $\tan -780^\circ =$ _____
e. $\sec 390^\circ =$ _____	f. $\cot 180^\circ =$ _____	g. $\sin 240^\circ =$ _____	h. $\sec 120^\circ =$ _____
i. $\tan \left(\frac{7\pi}{6}\right) =$ _____	h. $\cos \left(\frac{5\pi}{3}\right) =$ _____	j. $\sin \left(\frac{7\pi}{2}\right) =$ _____	k. $\sec \left(\frac{-11\pi}{4}\right) =$ _____